Standard 6 Practice Set

Solve problems using different strategies, to verify and interpret results, and to draw conclusions

Problem solving strategies:

Work backwards	Find a pattern
Adopt a different point of view	Solve a simpler analogous problem
Consider extreme cases	Make a drawing
Intelligent guessing and testing	Account for all possibilities
Organize data	Logical reasoning

1. Place the numbers from 1 through 9 into the grid below so that the sum of each row, column, and diagonal is the same. (This is also known as a magic square.)

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2. In a room with 10 people, everyone shakes hands with everyone else exactly once. How many handshakes are there?

3. The sum of two numbers is 12 and the product of the same two numbers is 35. Find the sum of the reciprocals of the two numbers.

4. Find the sum of the first 20 odd numbers. How could you find the sum of the first n odd numbers?

5. Barry wants to get at least a B (80%) for the final grade in this course. So far he has taken four tests and earned the following grades: 60, 92, 88, 70. What grade will he have to make on the fifth test to get a final grade of B (80%)?

6. A treasure chest has been buried on a deserted square island that is surrounded by a shark-filled moat 20 feet wide. The sides of the island and the moat are made of concrete, not sand. You know that if you can just get on the island, you can find the treasure and be rich beyond your wildest dreams. You have two boards that are 19 feet long but no other tools. How can you get on the island?
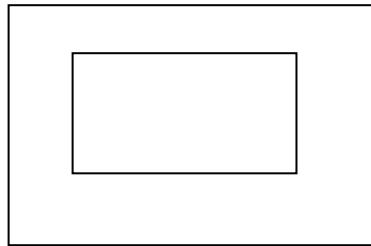
7. A farmer, a goat, a wolf, and a cabbage have to cross a river. A boat nearby only has enough room for the farmer and one other thing. What is the fewest number of trips he must take so that the goat does not eat the cabbage, and so the wolf does not eat the goat?

Answers for Standard 6 Practice Set

1. Since the sum of the numbers 1-9 is 45, each row must have a sum of 45/3 or 15. No extreme score could be in the center (9 or 8 or 2 or 1) because that would not allow for a sum of 15. The best guess for the center cell is 5. Now we look for pairs that add up to 10…

	1	
	5	
	9	

Leads to

8	1	6
	5	
	9	

And eventually a solution:

$$\begin{array}{ccc} 8 & 1 & 6 \\ 3 & 5 & 7 \\ 4 & 9 & 2 \end{array}$$

To solve this problem, you probably used some intelligent guessing and checking, some logical thinking -- in fact, you may have used many of the problem solving strategies listed at the top of the practice set.

2. To solve this problem, you may want to draw a circle that contains 10 people… or you may want to use a chart or a list like this (let the first person be A, the second B, …, the 10$^{\text{th}}$ J):

A B	B C	C D	D E	E F	F G	G H
A C	B D	C E	D F	E G	F H	G I
A D	B E	C F	D G	E H	F I	G J
A E	B F	C G	D H	E I	F J	
A F	B G	C H	D I	E J		H I
A G	B H	C I	D J			H J
A H	B I	C J				
A I	B J					I J
A J						

Total handshakes = 45

3. Make a list of numbers that add up to 12, along with the reciprocals and their products.

1$^{\text{st}}$ number	2$^{\text{nd}}$ number	Sum (12)	Product
1	11	12	11
2	10	12	20
3	9	12	27
4	8	12	32
5	**7**	**12**	**35**
6	6	12	36

Then the numbers we are looking for are 5 and 7: $5 + 7 = 12$ and $5 * 7 = 35$. Now we need to find the sum of the reciprocals of the numbers by finding a least common denominator (in this case, it is 35):

$$\frac{1}{5} \quad = \quad \frac{7}{35} \qquad\qquad \frac{7+5}{35} \quad = \quad \frac{12}{35}$$

$$+ \quad \frac{1}{7} \quad = \quad \frac{5}{35}$$

So the solution to the problem is 12/35.

4. Find the sum of the first 20 ODD numbers:

$1 + 3 + 5 + 7 + 9 + 11 + 13 + 15 + 17 + 19 + 21 + 23 + 25 + 27 + 29 + 31 + 33 + 35 + 37 + 39 =$

400

How can we use this information to find the sum of the first n odd numbers?

One way is to find a pattern using a chart. Use the pattern to fill in the last two lines of the chart.

Numbers being added	Numbers added	Sum
1	1	1
1 + 3	2	4
1 + 3 + 5	3	9
1 + 3 + 5 + 7	4	16
1 + 3 + 5 + 7 + 9	5	25
1 + 3 + 5 + 7 + 9 + 11	6	36

 Did you notice a pattern in the columns titled "Numbers added" and the "Sum"? If we square the "Numbers added" column, we get the sum. So the sum of the first 20 odd numbers is 20^2, or 400.

Now extend that formula to the nth term. Then the sum of the first n odd numbers is n^2.

5. To determine how Barry can earn a B in the course, we could use guess-and-check by substituting test scores for x:

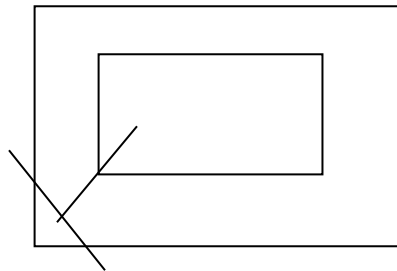$$\frac{60 + 92 + 88 + 70 + x}{5} = \frac{310 + x}{5} = 80$$

Or we could solve this algebraically by solving for x:

$$\frac{310 + x}{5} = 80 \qquad \textit{multiply both sides by 5}$$

$$310 + x = 400 \qquad \textit{now subtract 310 from both sides}$$

$$x = 90 \qquad \text{This is the answer!!}$$

6. Here we will use some creative thought… One way to get to the island is to place one board over the other (forming a T), like this:

7. One way to approach this problem is to draw the action… (let F = farmer, G = Goat, W = wolf, and C = cabbage)

	Left shore	River		Right shore
Begin	F, G, W, C			
1st trip	W, C	F, G	→	G
Return		F		
2nd trip	C	F, W	→	W, G
Return		F, G		
3rd Trip	G	F, C	→	W, C
Return		F		
4th Trip		F, G	→	W, C, G

This is only one possible sequence of events… you may have made deliveries in a different order.