Proportional Reasoning

Proportional thinking involves the ability to understand and compare ratios, and to predict and produce equivalent ratios. It requires comparisons between quantities and also between the relationships between quantities. It involves quantitative thinking as well as qualitative thinking. It is not dependent on a skill with a mechanical or algorithmic procedure. Therefore, this material does not simply show the cross multiply method and assume students can do proportional thinking. Time is spent exploring proportions from a variety of perspectives: equivalent fractions, multiplication by unit rates, graphing, as well as the cross multiply method. A feature of proportional situations is the multiplicative relationship among the quantities. The relationship is explored with tables, graphs, and equivalent fractions.

Proportional thinking involves problems of the following type:

1. John has canaries and parrots. For every 4 canaries he has 3 parrots. If all the canaries that John has are 28 how many parrots does he have?
2. To make Italian dressing you need 3 parts of vinegar for 8 parts of oil. How much vinegar do you need for 96 ml of oil?
3. At a fruit stand, 3 apples cost 90 cents. You want to buy 7 apples. How much do you have to pay?
4. George used 15 cans of paint to paint 18 chairs. How many chairs will George paint using 25 cans of paint?

Answers for these problems can be found at the end of this document.

Spatial Reasoning

Spatial reasoning is a process of forming ideas through the spatial relationships between objects. Geometry is an example of spatial reasoning at work. Graphs and maps present data in a format suitable for spatial reasoning.

Spatial intelligence might be one of less familiar kind of intelligence; however it has wide implications in many academic and professional disciplines. It is extremely important in disciplines such as mathematics and computer science. Spatial Intelligence also accounts for the thinking process of engineers, architects, designers, sculptors and inventors.

This is a problem that requires spatial thinking:

1. A is on the right of B
2. C is on the left of B
3. D is in front of C
4. E is in front of B.

What is the relationship between D and E?

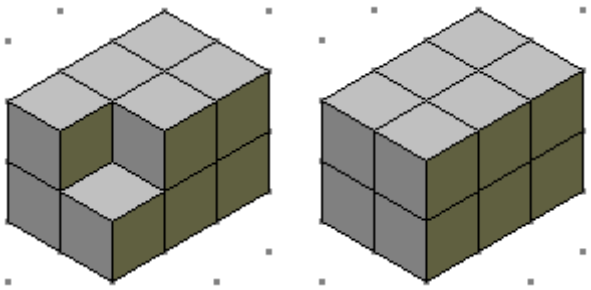
(Answers can be found at the end of this document)

The National Council of Teachers of Mathematics (NCTM, www.nctm.org) provides many examples of problems that require spatial reasoning:

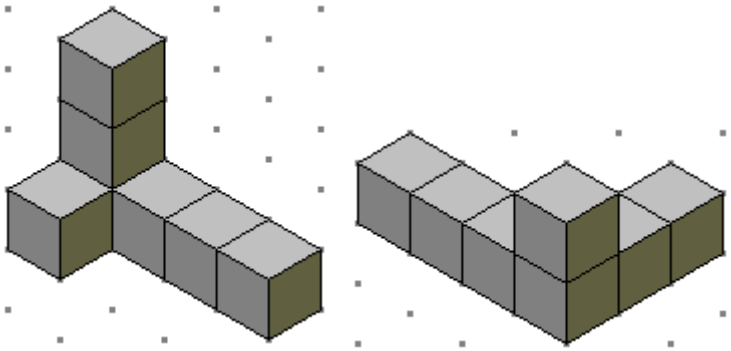
In this task, students will determine when two isometric drawings can represent the same shape.

Project the following images for the students to see:

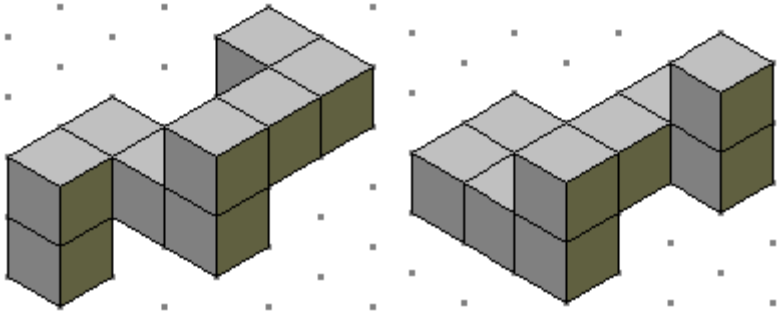
A.



B.

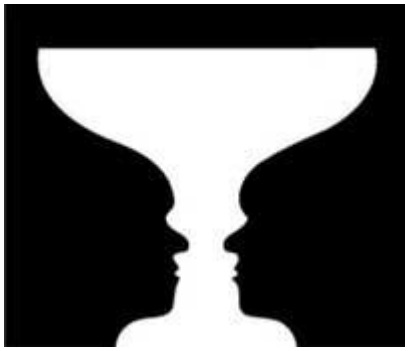


C.



Tilings, tessalations, and origami are examples of classroom activities that teachers can use to help students develop visualization and spatial reasoning.

Optical illusions are forms of spatial reasoning. Can you identify the illusion in the following artwork?



Faces or vases?

M. C. Escher, *Waterfall,* 1961

Valid and Invalid Arguments

For example, consider these two arguments:

All tigers are mammals.
No mammals are creatures with scales.
Therefore, no tigers are creatures with scales.

All spider monkeys are elephants.
No elephants are animals.
Therefore, no spider monkeys are animals.

These arguments share the same form:

All A are B;
No B are C;
Therefore, No A are C.

All arguments with this form are valid. Because they have this form, the examples above are valid. However, the first example is sound while the second is unsound, because its premises are false. Now consider:

All basketballs are round.
The Earth is round.
Therefore, the Earth is a basketball.

All popes reside at the Vatican.
John Paul II resides at the Vatican.
Therefore, John Paul II is a pope.

These arguments also have the same form:

All A's are F;
X is F;
Therefore, X is an A.

Arguments with this form are invalid. This is easy to see with the first example. The second example may *seem* like a good argument because the premises and the conclusion are all true, but note that the conclusion's truth isn't guaranteed by the premises' truth. It could have been possible for the premises to be true and the conclusion false. This argument is invalid, and all invalid arguments are unsound.

For more information, visit the following website.

http://www.iep.utm.edu/v/val-snd.html

Answers for problems within this document:

Proportional Reasoning

1. 21
2. 36
3. $2.10 or 210 cents
4. 30

Spatial Reasoning

What is the relationship between D and E?

D E

C B A

Can they match? A. yes B. no C. yes